39003

OTS: 60-41107

JPRS: 5214

3 August 1960

ON THE LARGE SCALE THERMO-HYDRODYNAMICAL PROCESSES IN A BAROCLINIC ATMOSPHERE

by G. I. Marchuk

-USSR-

Distributed by:

OFFICE OF TECHNICAL SERVICES
U. S. DEPARTMENT OF COMMERCE
WASHINGTON 25, D. C.

~~Price: $1.00~~

U. S. JOINT PUBLICATIONS RESEARCH SERVICE
205 EAST 42nd STREET, SUITE 300
NEW YORK 17, N. Y.

JPRS: 5214
CSO: 4614-N

ON THE LARGE-SCALE THERMO-HYDRODYNAMICAL PROCESSES IN A BAROCLINIC ATMOSPHERE

[This is a translation of an article by G.I. Marchuk, in Trudy, Inst. Fiz. Atmos., No 2, 1951; CSO: 4614-N]

The atmospheric thermodynamic-hydrodynamic processes can be condi tionally classified as large-scale and small-scale processes. The larg scale processes have characteristic horizontal dimensions of the order several thousand kilometers (extensive cyclones, anticyclones, pressure ridges and others), the processes of small-scale of one hundred kilomet The large-scale processes characterize the basic states of the atmosphe circulations, then against a background which depends on that, there oc the small-scale meteorological processes, which define the character of the weather in a particular area of the globe. As a rule the large-sca quasi-static processes manifest a known trait of stability and are able a certain sense to be the object of an independent investigation. The small-scale processes are more mobile (active), they are defined by pro of the large-scale and generally speaking, they are not able to be consi independently of the neighboring synoptic conditions.

For a study of the dynamics of atmospheric phenomena there is sho the necessity of constructing some mathematical theory which would most completely reflect the physical aspects of the matter. The laws of t hydrodynamics conformed to the atmospheric condition permit me to write corresponding system of differential equations, more or less completely describing the character and evolution of the field of meteorological elements. However for a long time, there was no success to mention wit equations of hydro-thermodynamics used directly to precalculate the

meteorological fields.

A step in the development of meteorological methods of prognosis of the weather was taken by I. A. Kibel in 1940 [2]. Introducing a characteristic scale for the investigation of synoptic processes, I. A. Kibel showed that such motions are quasi-geostrophic. This allowed him to rem what was non-essential to meteorology in the solution - the sound and gra waves. The idea of Kibel's on the construction of a theory of short-rang prognosis of the weather on the basis of the quasi geostrophic expressio was used subsequently by meteorological researchers. Thanks to a selection of the initial polytropic model, one succeeded in writing basic prognosic equations in a rather simple form either for the earth's surfac or for some level in the atmosphere.

In 1949 there was published a work of A. M. Obukhov [4], in which the author constructed an equation and gave his solution for the change o pressure with respect to the convergence of mass in a barotropic atmosphere.

In 1939 Rossby [9] and in 1940 Hurwitz [8] developed through a stu of the processes of large scale a method of small perturbations, using the well-known meteorological fact of the existence of a primary West-Ea flow of air. They obtained only the velocity of displacement of ~~air~~ an isolated harmonic-wave.

In 1943 there appeared the work of E. N. Blinova "The hydrodynamic theory of pressure waves and centers of action in the atmosphere" [1], i which the author succeeded in obtaining a solution of the general probl of weather prognosis, proceeding from the combined solution of the line ed system of hydrodynamic equations. The basic atmospheric centers of action were obtained.

The aim of the present work is a study of the ~~diffusion~~ processes

PROPOGATION

the disturbances at the level of the 1000 mb surface as well as at h levels, and with this to elucidate the interaction of the processes, that develop at different levels of a baroclinic atmosphere. Consid our basic problem the elucidation of the qualitative aspects of the mechanism of THE PROPAGATION of the disturbances we limited ourselves in t investigation to a study of large-scale processes only, for which th geostrophic model of the wind could be used with success.

The principal interest in the work is given to the deduction o equations that describe the surface process of PROPOGATION of a thermo disturbance with a consideration of the basic factors of a barocli mosphere. In the work questions related to the stability of the sol of the equations of thermo-hydrodynamics conforming to an atmospheri were considered; criteria for the stability of the solution of the e were obtained depending on the wave length of the disturbance, the c of the Coriolis parameter with latitude and the vertical profile of velocity of the West-East flow. The role of the factors of the hori macroturbulent exchange on the stability of atmospheric motion was c sidered. There was obtained criteria of the stability of the soluti the equations of hydrodynamics by the calculation of the factors of turbulent exchange.

In the present work we did not make it our aim to apply the re directly to the prognosis of weather. We consider this as a later problem, to be the subject of a serious study.

In conclusion we notice that we investigated in detail the case polytropic atmosphere. This was done in order that the simple quali aspect of the matter would not be hidden by the unwieldy calculations

Derivation of the Basic Equations

In the present paragraph we set forth the derivation of the basic equations of thermo-hydrodynamics of a baroclinic atmosphere.

We shall consider an x, y, z coordinate system where attached at the earth's surface, in such a way that the x axis is directed toward the east along a circle of latitude, the y axis along a meridian toward the earth, and the z axis is directed vertically upwards. In the adopted coordinate system the equations of motion of air masses in the atmosphere can be written in the form of Cromeko.

$$\text{(1)} \qquad \frac{\partial u}{\partial t} + v(\Omega + l) = -\frac{1}{\rho}\frac{\partial p}{\partial x} - \frac{\partial}{\partial x}\left(\frac{u^2+v^2}{2}\right)$$

$$\text{(2)} \qquad \frac{\partial v}{\partial t} - u(\Omega + l) = -\frac{1}{\rho}\frac{\partial p}{\partial y} - \frac{\partial}{\partial y}\left(\frac{u^2+v^2}{2}\right)$$

$$\text{(3)} \qquad g = -\frac{1}{\rho}\frac{\partial p}{\partial z}$$

where u, v are the components of the velocity vector along the x and y axis respectively; p is pressure; ρ is the density; g is the acceleration of the gravity force; $l = 2\omega \sin\varphi$ is the Coriolis parameter; ω is the angular velocity of the rotating earth, φ is the geographic latitude of place; $\Omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ is the vertical component of vorticity.

In equations (1)-(3) the terms that contain the vertical component of the velocity vector were excluded from the consideration for being values of a higher order of smallness. The system (1)-(3) is supplemented by the equation of continutiy

$$(4) \qquad \frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = 0$$

and the heat flow

$$(5) \qquad \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w(\gamma_a - \gamma) + \frac{\varkappa - 1}{\varkappa}\frac{T}{p}\left(\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y}\right) = \frac{\varepsilon}{c_p \rho}$$

where T is temperature

w is the vertical component of velocity

$\gamma = -\frac{\partial T}{\partial z}$

$\gamma_a = \frac{\varkappa - 1}{\varkappa}\frac{g}{C_p}$

ε heat flow to a unit volume in a unit time

$\varkappa = c_p / c_v$

c_p and c_v are the specific heat capacities of air for constant pressure and constant volume.

Let us differentiate (2) with respect to x, (1) with respect to y and subtract from the first the other. Then we obtain the Friedman's equation

$$(6) \qquad \frac{\partial \Omega}{\partial t} + u\frac{\partial \Omega}{\partial x} + v\frac{\partial(\Omega + l)}{\partial y} + (\Omega + l)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = \frac{1}{\rho^2}\left(\frac{\partial p}{\partial x}\frac{\partial \rho}{\partial y} - \frac{\partial p}{\partial y}\frac{\partial \rho}{\partial x}\right)$$

Eliminating from equation (6) the surface divergence by means of (4) and discarding in the result estimated terms of higher orders of smallness, we are left with the equation

$$(7) \qquad \frac{\partial \Omega}{\partial t} + u\frac{\partial \Omega}{\partial x} + v\frac{\partial(\Omega + l)}{\partial y} = \frac{l}{\rho}\frac{\partial \rho w}{\partial z}$$

With the aim of obtaining a formula, convenient for practical work, we shall pass from independent variables x, y, z, t to new variables x_1, y_1, p, t_1. Here the sign 1 by the new independent variables indicates the fact that the subscripted equality takes place on an isobaric surface $p =$ constant.

The transition from the original system of coordinates (x, y, z, t) to the new ones (x_1, y_1, p, t_1) is carried out by means of the transformations:

$$\frac{\partial}{\partial x_1} = \frac{\partial}{\partial x} - \frac{\partial p}{\partial x}\frac{\partial}{\partial p}, \qquad \frac{\partial}{\partial y_1} = \frac{\partial}{\partial y} - \frac{\partial p}{\partial y}\frac{\partial}{\partial p}$$

$$\frac{\partial}{\partial p} = -\frac{1}{g\rho}\frac{\partial}{\partial z} \qquad \frac{\partial}{\partial t_1} = \frac{\partial}{\partial t} - \frac{\partial p}{\partial t}\frac{\partial}{\partial p}$$

Further on the basis of estimations, one can show that the following expressions of correlation take place

$$\frac{\partial \Omega}{\partial t_1} = \frac{\partial \Omega}{\partial t}, \quad \frac{\partial \Omega}{\partial x_1} = \frac{\partial \Omega}{\partial x}, \quad \frac{\partial \Omega}{\partial y_1} = \frac{\partial \Omega}{\partial y}$$

As a result equation (7) may be written in the new coordinates as:

$$(8) \qquad \frac{\partial \Omega}{\partial t_1} + u\frac{\partial \Omega}{\partial x_1} + v\frac{\partial(\Omega + l)}{\partial y_1} = -gl\frac{\partial \rho w}{\partial p}$$

As indeed in the new coordinates the equation of statics takes the form:

$$\frac{\partial z}{\partial p} = -\frac{RT}{gp}$$

The equation of the heat flow in the new variables is written in the following form:

$$(10) \quad \frac{\partial T}{\partial t_1} + u\frac{\partial T}{\partial x_1} + v\frac{\partial T}{\partial y_1} + (\gamma_a - \gamma)\left[w - \left(\frac{\partial z}{\partial t_1} + u\frac{\partial z}{\partial x_1} + v\frac{\partial z}{\partial y_1}\right)\right] = \frac{\varepsilon}{c_p\rho}$$

In equations (8) and (10) we shall take the components of the velocity vector, u and v, in their geostrophic expressions:

$$u = -\frac{1}{l\rho}\frac{\partial p}{\partial y} = -\frac{g}{l}\frac{\partial z}{\partial y}; \quad v = \frac{1}{l\rho}\frac{\partial p}{\partial x} = \frac{g}{l}\frac{\partial z}{\partial x}$$

If, in addition, introducing into the discussion the variable ζ by means of the formula

$$\zeta = \frac{p}{P}$$

where $P =$ 1000 mb, then, assuming that the first and second derivative of $\frac{1}{l}$ are small by comparison with the other terms, we obtain the following system of equations:

$$\frac{\partial}{\partial t_1}\Delta z + \frac{g}{l}(z, \Delta z) + \beta\frac{\partial z}{\partial x} = -\frac{l^2}{P}\frac{\partial \rho w}{\partial \zeta} \tag{11}$$

$$\frac{\partial T}{\partial t_1} + \frac{g}{l}(z, T) = -(\gamma_a - \gamma)\left(w - \frac{\partial z}{\partial t}\right) + \frac{\varepsilon}{c_p \rho} \tag{12}$$

$$\frac{\partial z}{\partial \zeta} = -\frac{RT}{g\zeta} \tag{13}$$

Thus, we define a complete system of differential equations for the determ tion of the functions z, T and W.

Next it follows to add to ~~the~~ this system by the problem of boundary con tions and initial data. As a condition at the earth's surface we use the condition of equating the vertical velocity to zero $W = 0$, at the second boundary relation there is the condition of equating to zero the

vertical mass flow at the top of the atmosphere.

In our adopted designation, the boundary conditions for the systems (11)-(13) shall be written as

(14) $$\rho W\Big/_{\rho=1} = 0 \quad , \qquad \rho W\Big/_{\rho=0} = 0$$

For initial data it can be given for example, as

(15) $$z\Big/_{t_1=0} = f(x_1, y_1, \rho)$$

From equation (14) there can be obtained ρW :

(16) $$\rho W = -\frac{\rho}{\ell^2}\int_{\rho}^{1}\left\{\frac{\partial \Delta z}{\partial t} + \frac{g}{\ell}(z, \Delta z) + \beta\frac{\partial z}{\partial x_1}\right\} d\rho^{1}$$

Substituting (16) into (12) we shall eliminate the vertical veloc from the equation of heat flow.

In the resulting equations (12) and (13) we could find z and if the second of the boundary conditions (14) could be explicitly expre through z and T . It turns out that it is possible to do this.] this purpose we integrate (16) throughout the whole atmosphere, then we obtain the integral relation

(17) $$\int_{0}^{1}\left\{\frac{\partial}{\partial t_1}\Delta z + \frac{g}{\ell}(z, \Delta z) + \beta\frac{\partial z}{\partial x_1}\right\} d\rho = 0$$

which then will play the role of the missing boundary condition for th system (12), (13). We integrate the equation of statics (13)

1. An analogous formula was obtained by M. E. Shvets in 1941, [5].

$$z = z_0 + \frac{R}{g}\int_{\zeta}^{1} T\frac{d\zeta}{\zeta}$$

where z_0 is the height of the absolute topography of the 1000 mb surface above sea level. Now we obtain the system of differential equations for defining the functions z, z_0 and T

(18) $$\int_0^1 \left\{ \frac{\partial \Delta z}{\partial t_1} + \frac{g}{l}(z, \Delta z) + \beta \frac{\partial z}{\partial x_1} \right\} d\zeta = 0$$

(19) $$\frac{\partial T}{\partial t_1} + \frac{g}{l}(z, T) =$$
$$= -(\gamma_a - \gamma)\left[-\frac{P}{l^2 \zeta}\int_{\zeta}^{1}\left\{\frac{\partial \Delta z}{\partial t_1} + \frac{g}{l}(z, \Delta z) + \beta\frac{\partial z}{\partial x_1}\right\} d\zeta - \frac{\partial z}{\partial t_1}\right] + \frac{\varepsilon}{c_p \rho}$$

(20) $$z = z_0 + \frac{R}{g}\int_{\zeta}^{1} T\frac{d\zeta}{\zeta}$$

which then we assume as the basis of our subsequent considerations.

Linearization of the system of Equations

The system of differential equations (18)-(20) we consider in the first approximation in the following form:

(21) $$\int_0^1 \left\{ \frac{\partial \Delta z}{\partial t} + \left(\beta + \frac{g}{l}\frac{\partial \Delta z}{\partial y}\right)\frac{\partial z}{\partial x} - \frac{g}{l}\frac{\partial z}{\partial y}\frac{\partial \Delta z}{\partial x} \right\} d\zeta = 0$$

(22) $$\frac{\partial T}{\partial t} - \frac{g}{l}\left\{\frac{\partial T}{\partial x}\frac{\partial z}{\partial y} - \frac{\partial T}{\partial y}\frac{\partial z}{\partial x}\right\}$$

(23) $$z = z_0 + \frac{R}{g}\int_{\zeta}^{1} T\frac{d\zeta}{\zeta}$$

Here and everywhere else that follows, the index 1 after the independent variables will be dropped.

As initial data for the system (21)-(23) we shall take the $z[x, y, \zeta, 0]$ field:

(24) $$z\big|_{t=0} = f(x, y, \zeta)$$

We linearize the system (21)-(23) with respect to the speed of the west-e flow. We let

$$z = \bar{z} + \frac{\partial \bar{z}}{\partial y} y + z'(x, y, \zeta, t)$$

$$T = \bar{T} + \frac{\partial \bar{T}}{\partial y} y + T'(x, y, \zeta, t)$$

where $\bar{z}$, $\bar{T}$, $\frac{\partial \bar{z}}{\partial y}$ and $\frac{\partial \bar{T}}{\partial y}$ are known functions of ζ

So far we have assumed by this that the zonal fields $\bar{z}$ and $\bar{T}$ sati the equation of statics

$$\frac{\partial \bar{z}}{\partial \zeta} = -\frac{R}{g} \frac{\bar{T}}{\zeta}$$

Then from this equation a relation is developed between $\bar{z}$ and $\bar{T}$ and $\frac{\partial \bar{z}}{\partial y}$ and $\frac{\partial \bar{T}}{\partial y}$,

(25) $$\bar{T} = -\frac{g}{R} \zeta \frac{\partial \bar{z}}{\partial \zeta},$$

(26) $$\frac{\partial \bar{T}}{\partial y} = -\frac{g}{R} \zeta \frac{\partial}{\partial \zeta}\left(\frac{\partial \bar{z}}{\partial y}\right)$$

Substituting (25) and (26) into (21)-(23) and neglecting the smaller quan tities of record order, we obtain the following linear system of differe tial equations:

$$(27) \qquad \int_0^1 \left\{ \frac{\partial \Delta z'}{\partial t} - \frac{g}{l}\frac{\partial \bar{z}}{\partial y}\frac{\partial \Delta z'}{\partial x} + \beta \frac{\partial z'}{\partial x} \right\} d\rho = 0$$

$$(28) \qquad \frac{\partial T'}{\partial t} - \frac{g}{l}\left[\frac{\partial \bar{z}}{\partial y}\frac{\partial T'}{\partial x} + \frac{g}{R}\rho \frac{\partial}{\partial \rho}\left(\frac{\partial \bar{z}}{\partial y}\right)\cdot\left(\frac{\partial z'}{\partial x}\right)\right] = 0$$

$$(29) \qquad z' = z_0 + \frac{R}{g}\int_\rho^1 T' \frac{d\rho}{\rho}$$

For the complete determination of the problem we add to the linear system (26)-(28) the initial data

$$(30) \qquad z'\Big|_{t=0} = f(x, y, \rho)$$

The solution of the system (27)-(29) we shall seek in the followin manner: first we find the solution of the system (28), (29) for the initial condition (30), computing the known function $z_0'(x, y, t)$ and after that as the solution of (28), (29) will have been found, we sh. substitute it into the integral relation (27), from which we then determined the form of the boundary function $z_0'(x, y, t)$.

Considering (26) we obtain from the equation of statics

$$(31) \qquad T' = -\frac{g}{R}\rho\frac{\partial z'}{\partial \rho}$$

Subsequently the stroke over functions under consideration will be dropped.

Now we shall substitute (31) in the equation of heat influx (28). the result we arrive at the following differential equation of second

order for the determination of the function $z(x, y, \zeta, t)$:

$$\frac{\partial^2 z}{\partial t \partial \zeta} + U(\zeta) \frac{\partial^2 z}{\partial x \partial \zeta} - \frac{dU}{d\zeta} \frac{\partial z}{\partial x} = 0 \tag{32}$$

Here we have introduced the definition

$$U = -\frac{g}{l} \frac{\partial \bar{z}}{\partial y}$$

The function $U(\zeta)$ represents the velocity of the west-east flow. In diagram 1 is shown the average climatic value of the function $U(\zeta)$ fo winter. In equation (32) we shall change to a new independent variable b means of the formula $U = U(\zeta)$. Then the equation (32) take s the fo

$$\frac{\partial^2 z}{\partial U \partial t} + U \frac{\partial^2 z}{\partial U \partial x} - \frac{\partial z}{\partial x} = 0 \tag{33}$$

The initial and boundary conditions of equation (33) will be:

$$\left. \begin{aligned} z\big|_{t=0} &= f_1(x, y, U)^{1}, \\ z\big|_{\zeta=1} &= z_0(x, y, t) \end{aligned} \right\} \tag{34}$$

The solution of equation (33) we shall seek in the following manner: we differentiate by U both parts of equation (33); AS A result we obta a differential equation of third order,

$$\frac{\partial^3 z}{\partial U^2 \partial t} + U \frac{\partial^3 z}{\partial U^2 \partial x} = 0. \tag{35}$$

LET $\partial^2 z / \partial U^2 = z_1$

1 Subsequently for the sake of simplicity $f_1(x, y, U)$ we shall designate by $f(x, y, U)$

The function z_1 must satisfy the differential equation of first order

(36) $$\frac{\partial z_1}{\partial t} + U\frac{\partial z_1}{\partial x} = 0$$

The total integral of equation (36) has the form

$$z_1 = \psi(x - Ut, y, U)$$

where ψ is an arbitrary function of the parameters and the expressi[on] $\overline{x} = x - Ut$. Hence

$$\frac{\partial^2 z}{\partial U^2} = \psi(\overline{x}, y, U).$$

We consider the following facts:

$$\left.\frac{\partial^2 z}{\partial U^2}\right|_{t=0} = \psi(x, y, U)$$

but from another aspect, by condition (34) there follows that

$$\left.\frac{\partial^2 z}{\partial U^2}\right|_{t=0} = \frac{\partial^2 z}{\partial U^2}(x, y, U)$$

Therefore

$$\psi(x - Ut, y, U) = \frac{\partial^2 f}{\partial U^2}(\overline{x}, y, U)$$

where the differentiation is carried out only with respect to the paramet[er] U, but not with $\overline{x} = x - Ut$. As a result we arrive at the equation

$$\frac{\partial^2 z}{\partial U^2} = \frac{\partial^2 f}{\partial U^2}(\overline{x}, y, U)$$

the integral of which, satisfies equation (33), readily found without d[ifficulty].

It has the form

(38) $$z = \phi(x,y,t) + U\int_0^t \frac{\partial\phi}{\partial x}\,dt + \int_{U(1)}^{U} (U-U')\,\frac{\partial^2 f(\bar{x}',y,U')}{\partial U'^2}\,dU' + U\frac{\partial f(x,y,U)}{\partial U}\Big/_{\rho=1}$$

where $\phi(x,y,t)$ is an arbitrary function, $\bar{x}' = x - U't$ and $U(1) = U\big|_{\rho=1}$. The form of the arbitrary function is determined by the second boundary condition (34) but first we shall turn our attention to the following circumstances. Up till now we did not fix the form of the arbitrary function $U(\rho)$, but we considered it arbitrary. By this it was privately assumed that $\rho = \rho(U)$ is a simple [single valued or ORC] function of its argument (where $\rho(U)$ is the inverse function of $U(\rho)$). However in real atmospheric conditions the function $\rho(U)$, generally speaking, is a multi-valued function. It is natural therefore that (38) cannot give the solution of the problem in all of the interval $0 \leq \rho \leq 1$.

It is easy to be convinced perhaps that the difficulty mentioned could be easily overcome by a simple method. In the solution (38) we return to the old independent variable ρ. As a result we obtain the solution of the problem good for the whole interval of the change of

$$z = \phi(x,y,t) + U(\rho)\int_0^t \frac{\partial\phi}{\partial x}\,dt + \int_1^{\rho} [U(\rho) - U(\eta)]\,\frac{\partial^2 f(\bar{x}',y,U')}{\partial U'^2}\Big/_{U'=U(\eta)} \times U_\eta\,d\eta + U(\rho)\frac{\partial f}{\partial U}\Big/_{\rho=1}$$

where $U_\eta = \dfrac{\partial U}{\partial \eta}$.

If now to take into account that, as a rule, $U(1) = 0$,[2]

[2] Strictly speaking, $U(1) = U_0$ is a very small value. Therefore it can be considered that $U(1) = 0$ approximately although this suggestion is not essential, for one can from the very first consider $U(1) = U_1 \neq 0$.

then from the limiting condition (34) it follows $\phi(x,y,t) = z_0(x,y,t)$.

Thus, the solution of the problem will have the form

$$(39) \quad z = z_0(x,y,t) + U(\xi)\int_0^t \frac{\partial z_0}{\partial x}\,dt + \int_1^\xi [U(\xi)-U(\eta)] \times \frac{\partial^2 f(\bar{x},y,U')}{\partial U'^2}\Big|_{U'=U'(\eta)} U_\eta\, d\eta + U(\xi)\frac{\partial f}{\partial U}\Big|_{\xi=1}$$

We shall recall that we assumed z_0 is a known function of the coordinates and time, although in fact its form is not completely known to us.

It was mentioned above that the form of the function $z_0(x,y,t)$ could possibly be determined by the integral relation (27). For this purpose we substitute (39) under the integral sign in (27) and we shall carry out the necessary operations of differentiation and integration. Then for the definition of z_0 we obtain the following integro-differential equation:

$$(40) \quad \left(\frac{\partial}{\partial t} + 2\bar{U}\frac{\partial}{\partial x}\right)\Delta z_0 + \beta\frac{\partial z_0}{\partial x} + \bar{U^2}\int_0^t \frac{\partial^2 \Delta z_0}{\partial x^2}\,dt + \beta\bar{U}\int_0^t \frac{\partial^2 z_0}{\partial x^2}\,dt = -\int_0^1 R\left\{\int_1^\xi [U(\xi)-U(\eta)]\frac{\partial^2 f(\bar{x},y,U')}{\partial U'^2}\Big|_{U'=U'(\eta)} U_\eta\, d\eta + U(\xi)\frac{\partial f}{\partial U}\Big|_{\xi=1}\right\} d\xi$$

where

$$\bar{U} = \int_0^1 U(\xi)\,d\xi, \quad \bar{U^2} = \int_0^1 U^2(\xi)\,d\xi$$

$$R = \Delta\left(\frac{\partial}{\partial t} + U(\xi)\frac{\partial}{\partial x}\right) + \beta\frac{\partial}{\partial x}$$

We differentiate (40) by t. Then we obtain the fundamental equation for the perturbations of z_0

$$(41)\qquad \left(\frac{\partial^2}{\partial t^2} + 2\bar{U}\frac{\partial^2}{\partial x\,\partial t} + \bar{U}^2\frac{\partial^2}{\partial x^2}\right)\Delta z_0 + \beta\frac{\partial}{\partial x}\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right) z_0 = \phi_t'$$

where $\phi' = \frac{\partial \phi}{\partial t}$ and

$$\phi = -\int_0^1 R\left\{\int_1^{\xi}\left[U(\xi) - U(\eta)\right]\frac{\partial^2 f(\bar{x}, y, U')}{\partial U'^2}\bigg|_{U'=U(\eta)} U_\eta\, d\eta + U(\xi)\frac{\partial f}{\partial U}\bigg|_{\xi=1}\right\} d\xi$$

The initial data for equation (41) will be:

$$(42)\qquad \begin{aligned} z_0\big|_{t=0} &= f(x, y, 0) \\ \frac{\partial \Delta z_0}{\partial t}\bigg|_{t=0} &= 2\int_0^1 \frac{\partial \Delta f}{\partial x}\,\xi\, U_\xi'\, d\xi - \beta\int_0^1 \frac{\partial f}{\partial x}\, d\xi \end{aligned}$$

For $t = 0$ the second of the relations (42) is an immediate result from (40).

In such a manner, the problem of the position of the disturbance of the heights of the 1000 mbs surface reduces to the integration of the differential equation (41) for the initial data (42). Before proceeding directly to the solution of the problem which has thus been developed, let us consider the following useful fact. In the case where the atmosphere both instantanesously and in its average state is polytropic i.e. when the temperature of the air particles with height falls according to a linear law:

$$(43)\qquad \begin{aligned} T &= T_0 - \gamma z \\ \bar{T} &= \bar{T} - \gamma z \end{aligned}$$

Equation (41) is considerably simplified and assumes a form

$$(44)\qquad \left(\frac{\partial^2}{\partial t^2} + 2\bar{U}\frac{\partial^2}{\partial x\,\partial t} + \bar{U}^2\frac{\partial^2}{\partial x^2}\right)\Delta z_0 + \beta\frac{\partial}{\partial x}\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right)z_0 =$$

This comes out of the following considerations: in view of the adopted hypothesis of polytropicity of the atmosphere, temperature and pressure are connected by the relation

$$\frac{T}{T_0} = \left(\frac{p}{P}\right)^{\kappa} = \varsigma^{\kappa}$$

where $\kappa = \frac{R\gamma}{g} \approx 0.175$ and $T_0 = T/_{\varsigma=1}$

Then the barotropic form will have the appearance

$$z = z_0 + \frac{R}{g}\int_{\varsigma}^{1} T\,\frac{d\varsigma}{\varsigma} = z_0 + \frac{RT_0}{g\kappa}\left(1-\varsigma^{\kappa}\right)$$

and

$$(45)\qquad \bar{z} = \bar{z}_0 + \frac{R}{g}\int_{\varsigma}^{1} \bar{T}\,\frac{d\varsigma}{\varsigma} = \bar{z}_0 + \frac{R\bar{T}_0}{g\kappa}\left(1-\varsigma^{\kappa}\right)$$

By means of using the second relation of (45) we obtain

$$(46)\qquad U = U_0 - \frac{R}{l\kappa}\frac{\partial \bar{T}_0}{\partial y}\left(1-\varsigma^{\kappa}\right)$$

where $U_0 = U(1)$. Eliminating from (46) and the first relati[on] of (45) the expression $\frac{R}{\kappa}\left(1-\varsigma^{\kappa}\right)$ we obtain:

$$(47)\qquad z = z_0 - \frac{l}{g}\,\frac{T_0}{\partial\bar{T}_0/\partial y}\left(U-U_0\right)$$

From (47) it follows that in the polytropic atmosphere the perturbations of the heights of the absolute topography of isobaric surfaces are a linear function of the relative velocity of the west-east flow, therefo[re] all derivatives of z by U above the first reduce directly to zero an[d] equation (41) takes the form (44).

The polytropic atmosphere, although favored for its simplicity, represents a highly crude model . EVIDENTLY incorrect if speaking for example of the stratosphere), therefore

we will conduct arguments for the general case of a baroclinic model of the atmosphere.

- The interdependence of the fields of meteorological elements on various atmospheric levels in the thermo-hydrodynamic process

One of the fundamental problems of theroetical meteorology is the investigation of the question concerning the interaction of hydrodynamic processes, developing at various levels of the atmosphere. The establishment of such an internal connection for the general baroclinic model of the atmosphere affords the possibility of determining the basic peculiarities of the mechanism of changes of pressure, temperature and vortex formation taking into account the kinematics and dynamics of atmospheric motions for this purpose. For an investigation of the present question, we shall proceed from an analysis of the solution of equation (40) of the preceding section.

The general solution of equation (40), which satisfies the initial data (42), valid for any moment of time, has a form so unwieldy and large for the determination of qualitative inquiries, that we, from the very beginning, must reject an investigation of it for our purposes, but at first we must go after a simpler and clearer method of analysis of the solution. Here the disucssion goes about such a transformation of equation (42), which would directly connect the hydrodynamical characteristics, which interest us, with the field of meteorolological elements in the whole thickness of the atmosphere. In the nature of

examined hydrodynamic quantities let us take, for example, the vertical component of vorticity, the principal part of which is in the geostrophic approximation

$$\Omega = \frac{g}{l} \Delta z$$

and the meridional component of vertical velocity

$$v = \frac{g}{l} \frac{\partial z}{\partial x}$$

Equation (42) can be transformed to the form

$$\frac{\partial \Omega_0}{\partial t} = -2 \int_0^1 d\zeta \int_0^{U(\zeta)} \frac{\partial \Omega}{\partial x} dU - \beta \int_0^1 v \, d\zeta \tag{48}$$

where

$$\Omega_0 = \Omega \big|_{\zeta = 1}$$

Let us obtain the first integral on the right side of (48) by parts. As a result we obtain:

$$\frac{\partial \Omega_0}{\partial t} = \int_0^1 \frac{\partial \Omega}{\partial x} L_1(\zeta) \, d\zeta + \int_0^1 v L_2(\zeta) \, d\zeta \tag{49}$$

where $L_1(\zeta) = 2\zeta \dfrac{\partial U}{\partial \zeta}$, $L_2(\zeta) = -\beta = \text{constant}$ are the influence functions of the corresponding hydro-dynamic factors $\dfrac{\partial \Omega}{\partial x}$ and v .

It is immediately clear, that the influence function $L_1(\zeta)$ serves as a function of height and therefore characterizes the contribution of disturbances in the vorticity field at various levels of the atmosphere to the local change of the vorticity field at the earth's surface.

First of all, we are interested in the question of the qualitativ

character, therefore here and everywhere in the future we shall consider the specific distribution of the west-east advection velocity, more characteristic of the temperate latitudes of the earth (one can, for example, study the climatic value of the west-east velocity of flow at latitude 50°N, figure 1). In figure 2 the influence function $L_1(\xi, \eta)$ is given.

The graph of the function $L_1(\xi, \eta)$ shows, that local time changes of vorticity at the 1000 mb surface depend chiefly on the character of meteorological elements in the lower and middle troposphere. For this we see, that disturbances in the vorticity field in the troposphere and lower stratosphere exert an opposite influence, with respect to sign, on local cyclogenesis at the earth's surface.

However, it follows that we turn out attention to the fact, that a direct evaluation of the series of quantities in equation (40) shows, that the effect of the factor of the deflecting force of the earth's rotation is important only for large scale disturbances.

Now let us consider the question concerning the local change of vorticity at any level of the atmosphere. For this purpose let us take advantage of the solution (39), with the help of which we shall obtain the equation for vorticity at any atmospheric level

$$\Omega = \Omega_0 + U\int_0^t \frac{\partial \Omega_0}{\partial x}\, dt + \int_1^{\xi} \left[U(\xi) - U(\eta)\right] \times \frac{\partial^2 \omega(\bar{x}, y, U')}{\partial U'^2}\Bigg|_{U' = U(\eta)} \frac{dU}{d\xi}\, d\eta + U \frac{\partial \omega(x, y, U)}{\partial U} \tag{50}$$

Let us differentiate (50) with respect to t_1, then let us put $t_1 = 0$ and let us use equation (48). As a result we obtain:

$$(51) \qquad \frac{\partial \Omega}{\partial t} = -U\frac{\partial \Omega}{\partial x} + \int_0^1 \frac{\partial \Omega}{\partial x} L_3(\xi, \eta)\, d\eta + \int_0^1 \nu L_2(\eta) \ldots$$

where $$L_3(\xi, \eta) = 2\left[\eta - \delta(\xi, \eta)\right]\frac{dU}{d\eta}$$

while $$\delta = \begin{cases} 1, & \text{if } \eta > \xi \\ 0, & \text{if } \eta \leq \xi \end{cases}$$

The first term on the right side of (51) $-U\frac{\partial \Omega}{\partial x}$ describes local cyclogenesis at the given level, at the expense of vorticity along latitude circles. The second and third terms describe the effect of dynamic factors, on the whole, connected with the redistribution of the energy of the disturbances by vertical currents from these atmospheric levels and others and by the meridional outbreaks of air masses.

The graph of the influence function $L_3(\xi, \eta)$ is given in figure (3).

Before turning to the discussion of the graph of the influence function $L_3(\xi, \eta)$, let us carry out some investigations, concerning the individual time derivative of the vorticity. Making use of expression (51), let us form the complete derivative of vorticity $d\Omega/dt$.

It is immediately clear, that in the linear approximation the principal part of the individual derivative, with respect to time, of the

vorticity is presented in the following form:

$$\frac{d\Omega}{dt} = \frac{\partial\Omega}{\partial t} + U\frac{\partial\Omega}{\partial x},$$

then from (51) we obtain:

$$\frac{d\Omega}{dt} = \int_0^1 \frac{\partial\Omega}{\partial x} L_3(\xi,\eta)\,d\eta + \int_0^1 V L_2(\eta)\,d\eta \tag{52}$$

Analysis of the function $L_3(\xi,\eta)$ shows, that since the second term in (52) does not depend on height, for which the calculation of $d\Omega/dt$ was derived, then the change of the individual derivative of vorticity, from level to level in the atmosphere, will be characterized just by the influence function $L_3(\xi,\eta)$.

The form of the influence function L_3 shows, that the solitary concentrated influence of the substance $\partial\Omega/\partial x$ at a given leve[l] in the troposphere gives rise to local changes of the vorticity field of one sign in the layers, situated below this level and also in the stratosphere, and of opposite sign in the tropospheric layers above the considered level.

Let us turn to the consideration of vertical currents in the atmosphere.

Vertical currents are a component part of the mechanism of the general circulation of the atmosphere. With the help of vertical curren[ts] the redistribution of air masses, kinetic and potential energy from one level to another takes place, depending on the specific distributio[n] of the thermobaric fields throughout the thickness of the atmosphere.

For the calculation of the vertical currents let us take advantag[e]

of Friedman's equation, written in the following form:

(53) $$\frac{d\Omega}{dt} + \beta v = -\frac{lg}{P}\frac{\partial \rho w}{\partial \zeta}$$

and let us integrate it with respect to ζ with the boundary condition

$$\rho w \big/_{\zeta=1} = 0$$

Then we obtain

(54) $$\rho w = \frac{P}{lg}\int_{\zeta}^{1}\left\{\frac{d\Omega}{dt} + \beta v\right\} d\zeta$$

but from (52) it follows, that

(55) $$\frac{d\Omega}{dt} = \int_{0}^{1}\frac{\partial\Omega}{\partial x} L_3(\zeta,\eta)\,d\eta + \int_{0}^{1} v L_2(\eta)\,d\eta$$

Therefore substituting (55) into (54) and carrying out the integration with respect to ζ, we obtain:

(56) $$g\rho w = \frac{P}{l}\left\{ Z\int_{0}^{1}\frac{\partial\Omega}{\partial x} L_4(\zeta,\eta)\,d\eta + \int_{0}^{1} v L_5(\zeta,\eta)\,d\eta\right\}$$

where $L_4 = [\eta(1-\zeta) + (\zeta-\eta)\delta]\frac{dU}{d\eta}$

$L_5 = \beta[\delta - (1-\zeta)]$.

From the figure (4) it follows, that the greatest vertical occurs in the neighborhood of the level, to which the point influence of the substance $\partial\Omega/\partial x$ was applied, while in the strkatospheric and tropospheric layers the vertical currents will have the opposite sign.

The graph of the function $L_5(\zeta,\eta)$ (figure 5) shows, that

the perturbation in the meridional component of the vector velocity at the given level gives rise to vertical currents of opposite signs above and below this level.

Now let us consider the question concerning, to what extent our mathematical construction reflects the physical picture of the advection of disturbances. For this purpose, let us consider, in the nature of an initial field of heights of isobaric surfaces, a harmonic wave in a west-east advection field.

$$z = \bar{z}(\zeta) + \frac{l}{g}\left[U\cdot y + A(\zeta)\sin\lambda x\right] \tag{57}$$

where $\bar{z}(\zeta)$ the standard value of the heights of isobaric surfaces

$U =$ constant for the whole globe

$A(\zeta)$ amplitude factor, which, for its determination, we shall consider positive and damped when $\zeta \to 0$

$\lambda = \frac{2\pi}{L}$ where L — wave length of the disturbance.

Taking (57) into account, let us form the expression for Ω and $\partial\Omega/\partial x$ for $t = 0$. As a result we obtain:

$$\Omega\Big|_{t=0} = -\lambda^2 A(\zeta)\sin\lambda x$$
$$\frac{\partial\Omega}{\partial x}\Big|_{t=0} = -\lambda^3 A(\zeta)\cos\lambda x$$

Next, let us form the equation

$$\frac{\partial\Omega_0}{\partial t}\Big|_{t=0} = \int_0^1 \frac{\partial\Omega}{\partial x}\Big|_{t=0} L_1(\eta)\,d\eta\,\cos\lambda x = M_1\cos\lambda x \tag{58}$$

where

(59) $$M_1 = -\lambda^3 \int_0^1 A(\eta) L_1(\eta)\, d\eta$$

Here and everywhere in the future, for the sake of simplicity, let us consider, that for the tropospheric layers of the atmosphere $\frac{dU}{d\xi} < 0$, while $A(\xi)$ is a function, which is damped with height and $M_1 > 0$.

As for our unknown solution, it is found in a class of periodic functions χ, which, obviously, satisfy the relation

$$\frac{\partial \Omega}{\partial t} = \frac{g}{l} \frac{\partial \Delta}{\partial t} z = -\lambda^2 \frac{g}{l} \frac{\partial z}{\partial t}$$

Therefore

(60) $$\frac{g}{l} \frac{\partial z_c}{\partial t}\Big|_{t=0} = -\frac{M_1}{\lambda^2} \cos \lambda \chi$$

Analogously from (52) and (56) we obtain:

(61) $$\frac{d\Omega}{dt} = M_2(\xi) \cos \lambda \chi$$

(62) $$g \rho w = M_3(\xi) \cos \lambda \chi$$

where

$$M_2(\xi) = \lambda^3 \int_0^1 A(\eta) L_4(\xi, \eta)\, U'_\eta\, d\eta$$

$$M_3(\xi) = -\lambda^3 \int_0^1 A(\eta) L_5(\xi, \eta)\, U'_\eta\, d\eta$$

If we assume, that $A(\zeta)$ diminishes with height, while the scale of the perturbations has the order of 1000 kilometers, then $M_2(\zeta)$ and $M_3(\zeta)$ will, as a rule, also be positive functions in the whole troposphere. Let us turn to the analysis of formulas (58)-(62). Let us assume, that the point of observation is situated at the level of the 1000 mb surface $\zeta = 1$. Then from formulas (58)-(62) it follows, that in the front part of the pressure trough (c, d, e) (figure 6) the region of pressure fall is situated from the center b to point d.

This region coincides with the zone of increase of the individual derivative of vorticity $\frac{d\Omega_0}{dt} > 0$, i.e. with the zone of cyclogenesis. The region considered, is characterized by ascending vertical currents.

On the contrary, in the front part of the ridge of high pressure (a, b, c) the region of pressure rise is situated from the center to point b. The individual derivative $\frac{d\Omega}{dt}$ in this region is negative, therefore we have the case with the zone of anticyclogenesis. This zone is characterized by descending vertical currents. Analogous arguments can be supplied for other atmospheric levels.

So, we see, that our mathematical construction is found in agreement with the physical side of the matter.

After the qualitative analysis of the mechanism of pressure change has been carried out, it is possible to approach the question concerning the search for the quantitative relations with the help of the integration of the basic equation (41).

The solution of the equatinn for changes of pressure

If in the preceding paragraph the discourse proceeded mainly about the investigation of the qualitative structure of the process, then in the present paragraph we shall make it our aim to obtain the general solution of the equation

(63) $$\left(\frac{\partial^2}{\partial t^2} + 2\bar{U}\frac{\partial^2}{\partial x \partial t} + \bar{U}^2\frac{\partial^2}{\partial x^2}\right)\Delta z_0 + \beta\frac{\partial}{\partial x}\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right)z_0 = \phi_t'$$

with the initial data

$$\zeta_0\big|_{t=0} = f(x, y, 0)$$

(64)

$$\frac{\partial \Delta \zeta_0}{\partial t}\bigg|_{t=0} = 2\int_0^1 \frac{\partial \Delta f}{\partial x} \rho\, U_0'\, d\rho - \beta \int_0^1 \frac{\partial f}{\partial x}\, d\rho$$

We shall consider, that the earth is either flat or has the form of a cylinder, generated, parallel to the earth's axis. Further let us assume, that the initial distribution of the perturbation field can be persented in the form of an isolated harmonic wave

(65) $$f(x, y, \rho) = Re\, f_{mn}(\rho)\, e^{imx + iny}$$

where m and n are any numbers.

Such a presentation of the initial field is quite expedient for the reason, that by the superposition of a similar family of solutions for various wave numbers it is possible to construct a solution for any initial data.

We shall look for the solution of equation (63) in the form

(66) $$\zeta_0(x, y, t) = Re\, \zeta_{mn}(t)\, e^{imx + iny}$$

Substituting (66) and (65) into (63) and (64), we obtain the follo[wing] equation and initial data for the new unknown function $\zeta_{mn}(t)$:

(67) $$\left\{ \frac{d^2}{dt^2} + im\left[2\bar{U} - \frac{\beta}{r^2}\right]\frac{d}{dt} + (im)^2\left[\bar{U}^2 - \frac{\beta}{r^2}\bar{U}\right]\right\} = \frac{d\, \mathcal{J}_{mn}}{dt}$$

$$z_{mn}\big|_{t=0} = f_{mn}(0)$$

(68) $$\frac{dz_{mn}}{dt}\Big|_{t=0} = im\left[\left(\frac{\beta}{r^2} - 2\bar{U}\right) f_{mn}(0) - \right.$$
$$\left. - \left(\bar{U}^2 - \frac{\beta}{r^2}\bar{U}\right) f'_{mn}(0)\right] - \phi_{mn}(0)\big|_{t=0}$$

where

$$r^2 = m^2 + n^2, \quad f'_{mn}(0) = \frac{df_{mn}}{dU}\Big|_{U=0}$$

while

$$\phi_{mn}(t) = \int_0^1 \left\{\frac{d}{dt} + im\left(U - \frac{\beta}{r^2}\right)\right\} \int_1^{U(\rho)} \left[U(\rho) - \right.$$
$$\left. - U(\eta)\, \frac{\partial^2 f_{mn}(U)}{\partial U^2}\Big|_{U=U(\eta)} e^{-imU(\rho)t} \frac{dU}{d\eta}\, d\eta\, d\rho\right.$$

It does not offer any difficulty to integrate equation (67) for the initial data (68), and the result can be written in closed form:

(69) $$z_{mn}(t) = \frac{1}{q}\left\{q f_{mn}(0) \cos mqt - i\left[p f_{mn}(0) + \right.\right.$$
$$\left.\left. + (p^2 - q^2)\frac{df_{mn}}{dU}\Big|_{U=0}\right] \sin mqt\, e^{-impt} + \right.$$
$$+ i\,\phi_{mn}(0)\, \frac{e^{im\sigma_1 t} - e^{im\sigma_2 t}}{2m} - \int_0^t \frac{d\phi_{mn}}{d\tau} e^{-imp(t-\tau)} \sin q(t-\tau)\, d\tau$$

where $$p = \bar{U} - \beta/2r^2, \quad q = \sqrt{\bar{U}^2 - \bar{U^2} + \left(\frac{\beta}{2r^2}\right)^2}$$
$$\sigma_{1,2} = p \pm q$$

Equation (69) consists of two parts. The first part

(70) $$z_{mn}^{\pi} = \frac{1}{q}\left\{q f_{mn}(0) \cos mqt - i\left[p f_{mn}(0) + \right.\right.$$
$$\left.\left. + (p^2 - q^2)\frac{df_{mn}}{dU}\Big|_{U=0}\right] \sin mqt\right\} e^{-impt}$$

serves as the solution of the problem for a polytropic atmosphere and the

second part

$$z^{\sigma}_{mn} = \frac{1}{q}\left\{ i\,\varphi_{mn}(0)\,\frac{e^{im\omega_1\tau} - e^{im\omega_2\tau}}{2m} - \int_0^t \frac{d\varphi_{mn}}{d\tau} e^{-imp(t-\tau)} \times \sin q(t-\tau)\,d\tau \right\} \tag{71}$$

depends on the deviations of the initial perturbations from the polytropic state; in other words z^{σ}_{mn} takes into account the initial vertical baroclinicity of the atmosphere as a source of new formations of atmospheric disturbances.

Let us turn to the analysis of the solution (69). In the case of a polytropic atmosphere, despite the presence of a slope, with respect to height, of the velocity of the east-west advection, the disturbances at all levels are advected along the x axis with one and the same velocity

$$U = \overline{U} - \frac{\beta}{2r^2} \tag{72}$$

In fact, z^{π}_{mn} has the follbwing form:

$$z^{\pi}_{mn} = A_{mn}(t)\, e^{-im\left[\overline{U} - \frac{\beta}{2r^2}\right]t}$$

where

$$A_{mn}(t) = f_{mn}(0)\cos mqt - i\left[\frac{p}{q} f_{mn}(0) + \frac{p^2-q^2}{q}\frac{df}{dU}\Big|_{U=0}\right]\sin mqt$$

Therefore

$$z^{\pi} = A_{mn}(t)\, e^{im\left[x - \left(\overline{U} - \frac{\beta}{2r^2}\right)t\right]} \tag{73}$$

So, we arrive at the conclusion, that in a polytropic atmosphere the

advection velocity of disturbances has a value, different from that obtained with the aid of the Rossby formula [9]. Let us recall, that Rossby obtained the formula for the advection velocity of a plane wave in the following form:

$$C = U - \frac{\beta}{\mu^2} \tag{74}$$

where U is the velocity of the east-west advection, for example at the 500 mb surface.

In latter years, by synoptics, it was observed [7], that the advection velocity of disturbances did not agree with the formula of Rossby. In order to obtain the necessary agreement with it, the regression coefficients a and b were introduced so, that

$$C = a\left(U - \frac{\beta}{\mu^2}\right) + b \tag{75}$$

where a and b were found as a result of the processing of statistical data.

Such an approach to the generalized Rossby formula brought the value of the velocity of transfer of the wave, obtained from formula (75), somewhat nearer to reality.

Concerning the fact, that the formula, we obtained, agrees somewhat better with observations, one can try, even so, by the same statistical data produced in [7], for the coefficients a and b From (73) it follows, that perturbations of small wave length are advected by the flow with the velocity of the mean advection $\overline{U}$. Therefore, it is quite natural to identify the mean level of the

atmosphere (the level, at which the velocity U coincides with the mean advection velocity $\bar{U}$) with the level of the "basic flow," established by the Soviet meteorologist N. I. Trotsky[1].

The stability of atmospheric motion

The question of the stability of atmospheric motion has already been an object of investigation. In the first place, it follows to point out the work of N. E. Kochin [3] concerning the stability of Margules surfaces.

N. E. Kochin showed, that for some interval of wave lengths, the non-zonal oscillations of Margules surfaces could become unstable. This permitted N. E. Kochin to connect the mathematically established fact, by means of observations in daily synoptic practice, with cyclonic formations in atmospheric frontal zones. Thus, one of the important questions of meteorology -- frontal cyclogenesis -- found its further development in the works of N. E. Kochin.

The problem of the stability of large scale atmospheric

---

[1] It follows to have it in the form, given by N. I. Trotsky, that the velocity of the basic flow is not constrained to be directed toward the east, but can have an arbitrary direction, corresponding to the direction of the isobars in the mean troposphere.

disturbances in the zone of a predominant west-east wind was initiated in the work of J. Charney [6], in which the author established the fact of the existence of unstable solutions of the hydro-thermodynamic equations, which describe the atmospheric process relative to the quite considerable interval of perturbation wave length in the presence of a change of the west-east advection velocity U with height.

In the present section, we shall undertake the task of investigation of the question concerning the stability of the solutions of the equations of hydro-thermodynamics for a baroclinic model of the atmosphere, proceeding from an analysis of the solution (69).

Above it was found, that the qualitative side of the process of advection of perturbations in a baroclinic atmosphere is completely included in an examination of the process, which occurs in the conditions of a polytropic atmosphere, therefore for our deductions it is sufficient to consider the solution (73):

$$z^{II} = A_{mn}(t)\, e^{im\left[y - \left(\bar{U} - \frac{\beta}{2r^2}\right)t\right]} \tag{76}$$

where

$$A_{mn}(t) = f_{mn}(0)\cos mqt - i\left[\frac{p}{q} f_{mn}(0) + \frac{p^2 - q^2}{q}\frac{dF}{dU}\Big|_{U=0}\right]\sin mqt \tag{77}$$

From (76) it follows, that the stability, or instability of atmospheric motion is determined by the amplitude of the wave perturbation $A_{mn}(t)$. The analysis of (77) shows that the instability of the solution (76) can be shown for that case, when the expression

$$\sqrt{\left(\frac{\beta}{2r^2}\right)^2 - \left(\overline{U}^2 - \overline{U^2}\right)}$$

is a minimum value.

If the expression $\sqrt{\left(\frac{\beta}{2r^2}\right)^2 - \left(\overline{U}^2 - \overline{U^2}\right)}$ is real, then $A_{mn}(t)$ will be a function, bounded in time, in other words the solution will be stable. So, for the criterion of the stability of the solution of the equations of hydro-thermodynamics in the examined approximation, the following inequality serves:

$$\left(\frac{\beta}{2r^2}\right)^2 > \overline{U}^2 - \overline{U^2} \tag{78}$$

The left side of (78) is bound, by its origin, to the inertial factor -- the deflecting force of the earth's rotation. Therefore, it is already clear from the very beginning, that its effect is significant for large scale perturbations, which will be stabilized by the effect of the deflecting force of the earth's rotation, and not important for small scale perturbations, which in their development will depend on, foremost, the specific distribution of the large scale perturbations. If we recall that $\beta = \frac{2\omega_1 \sin\phi}{a_0}$, and for the sake of simplicity, we take $r^2 = m^2 = \left(\frac{2\pi}{L}\right)^2$, while m is a number, which shows how many times the length of the wave L goes along the latitude circle, then we have:

$$\left(\frac{\beta}{2r^2}\right)^2 = \frac{(\omega a_0)^2 \sin^6\theta}{m_1^4}$$

Hence we see, that $\left(\frac{\beta}{2r^2}\right)^2$ decreases exceptionally quickly with decrease in the dimensions of the perturbations, therefore, generally speaking, the inequality (76) is fulfilled up to m_1, approximately equal to 6 or 7. This means, that only the large planetary waves are stable, while all the rest are unstable, and the stability increases with decrease of latitude as a function of $\sin^6\theta$. Due to such a rapid decrease of $\left(\frac{\beta}{2r^2}\right)^2$ for the increase of m_1, it is possible to conclude, that small changes of the difference $\bar{U}^2 - \overline{U^2}$ show a comparatively small influence on changes of the region of stability of the solution.

In that case, when $\bar{U}^2 = \overline{U^2}$, i.e. the west-east advection velocity is constant with respect to height, all wave motions are stable. However it is quite clear, that such a case is never realized in nature.

It appears to us, that the linearization of our initial equations was essential. This led to the fact, that the amplitudes of the small scale perturbations in our examination, grew quite rapidly, which is found to be in obvious contradiction to reality.

The mentioned discrepancy tries to eliminate by the construction of such a model of atmospheric motions, that which would take into account the influence of the non-linear factors in the equations even in its very rough presentation -- in the form of dissipative (Диссипирущих) factors of macro-turbulent displacement, to the consideration of which we shall turn.

For this purpose, we shall proceed from the following system of equations of hydro-thermodynamics, analogous to (21)-(23):

(79) $$\int_0^1 \left\{ \frac{\partial \Delta z}{\partial t} + \beta \frac{\partial z}{\partial x} - \frac{g}{l}\frac{\partial \bar{z}}{\partial y}\frac{\partial \Delta z}{\partial x} - \nu \Delta\Delta z \right\} d\zeta = 0$$

(80) $$\frac{\partial T}{\partial t} - \frac{g}{l}\left[\frac{\partial \bar{z}}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial \bar{T}}{\partial y}\frac{\partial z}{\partial x}\right] - \nu \Delta T = 0$$

(81) $$z = z_0 + \frac{R}{g}\int_\zeta^1 T\,\frac{d\zeta}{\zeta}$$

where ν – the constant coefficient of turbulent exchange for the fields of vorticity and temperature,

z – deviations of the heights of isobaric surfaces from the mean zonal state

T – deviation of the temperature of air particles from the zonal distribution.

The system of differential equations (79)-(81) should not offer any difficulty to integrate, taking into account the boundary conditions and initial data, just as this was accomplished at preceding points of the present work; however for the purpose of investigating only the questions of the stability of the solutions it is sufficient to choose an easier way.

Let us assume, that the fields of meteorological elements z, T can be presented in the following form:

(82) $$z = z_{mn}(\zeta)\, e^{im(x-\sigma t)+iny}$$
$$T = T_{mn}(\zeta)\, e^{im(x-\sigma t)+iny}$$

Then so, that the function (82) should actually give the solu

of the system of equations (79)-(81), it is necessary to fulfill the following differential relationships, relating the functions z_{mn}, T_{mn}:

(83) $$\int_0^1 \left(\sigma - U + i\frac{\nu r^2}{m} + \frac{\beta}{r^2}\right) z_{mn}\, d\xi = 0$$

(84) $$\left(\sigma - U + i\frac{\nu r^2}{m}\right)\frac{\partial z_{mn}}{\partial U} + z_{mn} = 0.$$

Integration of equation (84) leads to the solution z_{mn} in the following form:

(85) $$z_{mn}(U) = z_{mn}(0)\left(\sigma - U + i\frac{\nu r^2}{m}\right)$$

For the determination of the unknown quantity σ, it follows to substitute (85) in (82) and to carry out the integration. As a result we arrive at the following algebraic equation for the determination of σ:

(86) $$\left(\sigma + i\frac{\nu r^2}{m}\right)^2 - \left(2\bar{U} - \frac{\beta}{r^2}\right)\left(\sigma + \frac{i\nu r^2}{m}\right) + \left(\bar{U}^2 - \bar{U}\frac{\beta}{r^2}\right) = 0$$

the solution of which has the form

(87) $$\sigma_{1,2} = -i\frac{\nu r^2}{m} + \left[\bar{U} - \frac{\beta}{2r^2} \pm \sqrt{\bar{U}^2 - \bar{U}^2 + \left(\frac{\beta}{2r^2}\right)^2}\right.$$

Analysis of the expression (87) shows, that in the nature of a criterion of stability for the solution of the hydro-thermodynamic

system of equations, taking intoaccount the horizontal macro-turbulent exchange, it is possible to take the following:

$$\nu r^2 > Re\, m \sqrt{\overline{U^2} - \overline{U}^2 - \left(\frac{\beta}{2 r^2}\right)^2} \tag{88}$$

A graph of the dependence of the region of stability on the parameters of the process and wave length has been carried out in figure 7. From figure 7 it follows, that the region of stability of the solution is decreased with an increase of $\frac{\overline{U^2} - \overline{U}^2}{\overline{U}}$, it being that the atmospheric perturbations of both the larger and smaller waves are stable.

If, for example, $\nu = 10^6 \frac{m^2}{sec}$,
$\overline{U^2} - \overline{U}^2 = 12 \frac{m^2}{sec^2}$, $\beta = 1.6 \times 10^{-11} \frac{1}{m\text{-}sec}$, then the perturbations having wave lengths greater than 4000 km and less than 2500 km, will be stable, while if the wave length of the distrubance is contained in the interval between 2500 km and 4000 km, then it will be unstable.

In conclusion let us note, that the results obtained relative to the instability of the solution, apparently, prove to be an investigation of the well known mathematical stylization of the problem. We imagine that a physically more complete formulation of the problem will permit it to be completely free from unstable solutions.

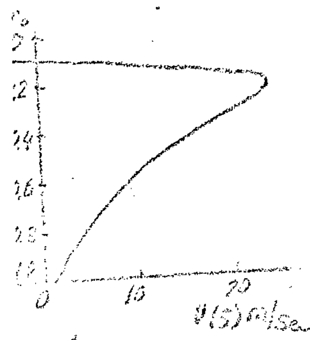


Fig. 1

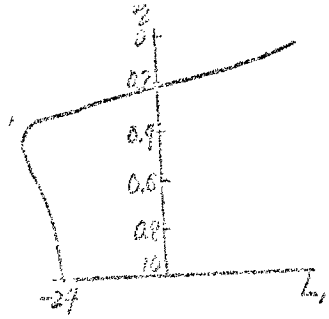


Fig. 2

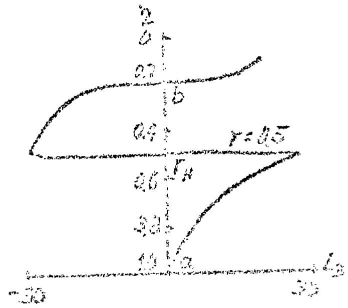


Fig 3

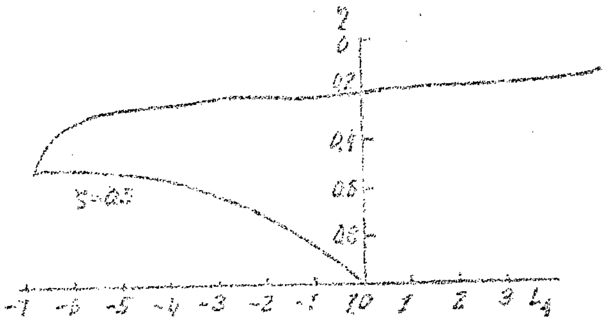


Fig. 4

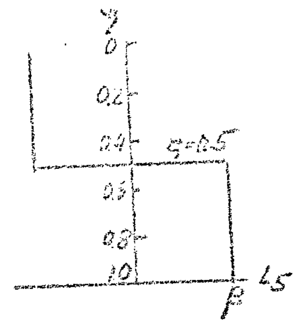


Fig. 5

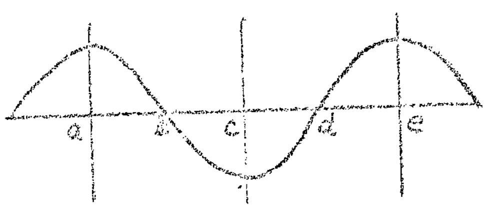
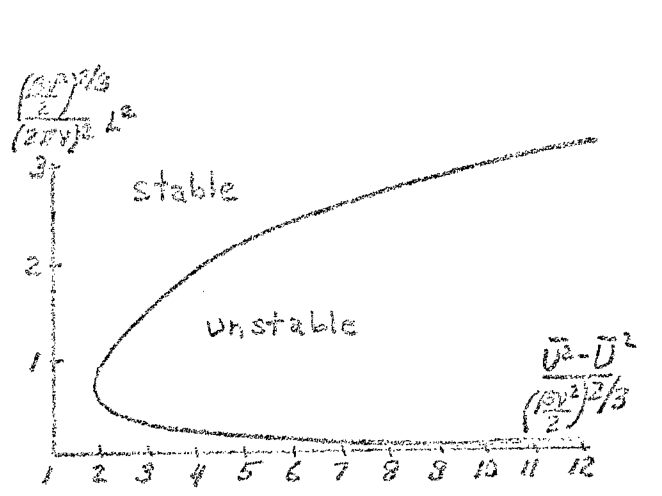


Fig. 6



REFERENCES

1. E.N. Blinova. Hydrodynamical theory of pressure waves, temperature waves and centers of action in the atmosphere. DAN, 39, no. 7, 1943.

2. I. A. Kibel. Application to meteorology of the equations of the mechanics of a baroclinic fluid. Izv. AN USSR, ser. geogr. i geofiz., no. 3, 1940.

3. N.E. Kochin and B.I. Izvekov. Dynamical Meteorology, chap. II, 1935.

4. A.M. Obukhov. Toward the question concerning the geostrophic wind. Izv. AN USSR, ser. geogr. i geofiz., no. 4, 1949.

5. M.E. Shvets. Determination of vertical velocities in moving air masses with the aid of the equations of hydromechanics. Izv. AN USSR, ser. geogr. i geofiz., no. 4-5, 1941.

6. J. Charney. The dynamics of long waves in a baroclinic westerly current. J. Meteor., 1947.

8. B. Haurwitz. The motion of atmospheric disturbances. J. Marine Research, 3, no. 3, 1940.

7. Dynamical methods in synoptic meteorology (Discussion), Q.J.R.M.S., 77, no. 333, 1951.

9. C.G. Rossby. Relation between variations in the intensity of the zonal circulation of the atmosphere and the displacements of the semi-permanent centers of action. J. Marine Research, 2, no. 1, 1939.

END